Multicarrier Bit-Loading in Presence of Biased Gaussian Noise Sources

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Abstract—Certain types of non-Gaussian noise sources in multicarrier communication systems behave effectively as modulating signals that control the first moment of the background Gaussian noise. The composite noise, which is the aggregate of the Gaussian and non-Gaussian noise, has a probability density function that is conditionally Gaussian with non-zero average, hence referred to as Biased-Gaussian. Impulsive interferers and timing-phase error are examples of such non-Gaussian noise sources. The BER-equivalent power of a composite noise source is defined as the power of a pure Gaussian noise source that yields the same bit-error rate (BER). The BER-equivalent noise for a Biased-Gaussian noise is simply the amplified version of the underlying Gaussian noise source. The amplification factor is derived from the characteristics of the non-Gaussian noise source. Any bit-loading algorithm designed for Gaussian noise sources is also applicable to Biased-Gaussian noise sources provided that the BER-equivalent SNR is used in place of the measured SNR.

Keywords- Multicarrier Communication; DSL; Bit-loading; Impulse Noise; Timing Phase Error

I. INTRODUCTION

In a multicarrier communication system, such as Asymmetric Digital Subscriber Line (ADSL) based on Discrete Multi-Tone (DMT), data is transmitted over a number of mutually independent sub-channels or tones. Each sub-channel carries only a certain portion of total data through Quadrature Amplitude Modulation (QAM) of the sub-carrier. The number of information bits loaded on each tone and the size of corresponding QAM constellation may potentially vary from one tone to another and depend generally on the relative power of signal and noise at the receiver. When the characteristics of signal and noise are known for all tones, a bit-loading algorithm can determine the optimal distribution of data bits and signal power amongst sub-channels.

The goal of bit loading is to find the right balance between three key quantities: data rate, transmission power and the probability of error. The algorithm usually optimizes for one of these quantities while the other two are fixed at their target values. This optimization is often constrained on conditions such as limited transmission power, power spectrum mask and integer number of bits per tone. The research on bit-loading has been fairly extensive with many proposed algorithms that have some variations on the optimization goals, constraints, and complexities [1-7]. These algorithms, however, mostly share the same model of additive white Gaussian (AWG) for the noise source. Although this model matches the characteristics of many noise sources, it is not quite applicable to some

important sources of interference and error. This paper addresses specifically two such sources: impulse noise and timing phase error.

A major source of impulse noise in ADSL systems is electric AC power switches and motors. For instance, an electric light dimmer switch with wires running in the proximity of the telephone wiring in a house is potentially a strong impulse interferer. ADSL systems are equipped with Reed-Solomon error correction coding to increase their resilience to impulse noise sources. However, the use of Reed-Solomon coding is optional. Some applications are sensitive to the interleaving latency often associated with Reed-Solomon coding and many ADSL lines are provisioned without that type of coding.

Timing mismatch and phase error is another source of impairment in multicarrier systems that may not be negligible in certain conditions. The effect of phase noise is more pronounced in high frequency sub-channels with large signal-to-noise ratio (SNR). In these sub-channels, the background noise is too small to mask the effect of other sources of error including phase noise.

The AWG noise model generally under-estimates the effects of impulse noise and phase error. Therefore, bit-loading based on AWG model results in high error rate when these noise sources are present. Another model that better matches these noise sources is Biased-Gaussian model, which is simply an additive, white Gaussian noise with non-zero average.

The rest of this paper presents an analysis of the error rate for Biased-Gaussian noise sources and its implications on bit-loading. It is shown how to apply this model to impulse noise and phase error and how to measure the parameters of the model. Although the problem is studied in the context of DMT in an ADSL system, the results are also applicable to any multicarrier system that allows flexible distribution of bits and power amongst sub-channels.

II. BIASED-GAUSSIAN NOISE MODEL

An ADSL transmitter modulates each sub-carrier with a data point in a QAM constellation. Because of the noise and other sources of interference, the receiver detects the data point with some distance from the transmitted constellation point. In order to avoid error in decoding data, the demodulated point should not pass the so-called *decision boundary* – the midway border between neighboring constellation points. Given the probability distribution of the noise, one can calculate the *minimum distance* between constellation points such that the

probability of error is less than some target value. When the power of noise is low, the minimum distance can be chosen to be small. This results in a denser constellation that carries more data bits.

Figure 1-a shows the probability density function of a one dimensional noise source. A decoding error occurs when the value of noise is larger than half the minimum distance. The probability of error is proportional to the area of the highlighted region. For a Gaussian noise source, this area is determined solely by the ratio of the minimum distance of constellation points, d , to the standard deviation of noise, σ . Therefore, in order to maintain a target probability of error, this ratio has to be set to a constant value as

$$\frac{d}{\sigma} = C \quad (1)$$

where C is a factor that depends on the target error rate and also coding gain and noise margin. For instance, for an error rate of 10^{-7} with no coding gain and noise margin, the value of C is close to 20.5 dB. Although this factor is also a function of constellation size, the relationship is fairly weak and is ignored in this analysis.

Similarly, when the Gaussian noise source is biased with a non-zero average of μ , the probability of error is dominated by the highlighted area shown in figure 1-b. In this case, the effective minimum distance, which is twice the distance between decision boundary and the center of distribution function, is smaller than the minimum distance of constellation and consequently the probability of error is higher than the case of unbiased noise. The effective minimum distance, d_{eff} , can be easily derived as

$$d_{eff} = d - 2\mu \quad (2)$$

To maintain a target error rate, the effective minimum distance has to satisfy equation (1) as follows

$$\frac{d_{eff}}{\sigma} = \frac{d - 2\mu}{\sigma} = C \quad (3)$$

It is insightful to define a BER-equivalent noise source for a biased Gaussian noise source as an unbiased Gaussian noise with the same error rate as the original biased noise. In order to obtain the same error rate the standard deviation of the equivalent noise source, σ_{eq} , has to satisfy the following

$$\frac{d}{\sigma_{eq}} = C \quad (4)$$

From equations (3) and (4), the standard deviation of the equivalent noise source is calculated from the parameters of the biased Gaussian noise source as

$$\sigma_{eq} = \left(1 + \frac{2\mu}{C\sigma}\right)\sigma \quad (5)$$

Fig. 1- Probability density functions of (a) an unbiased Gaussian, and (b) biased Gaussian noise sources. The highlighted area indicates the probability of receiving a data point out of the decision boundary, hence the probability of error.

This equation indicates that, for the sake of error rate analysis, a non-zero bias of a Gaussian source effectively amplifies the power of noise. Similarly, for the sake of bit-loading, a biased Gaussian noise source can be treated as an unbiased noise with the same standard deviation provided that the bit-loading algorithm uses the following additional noise margin term:

$$M = \left(1 + \frac{2\mu}{C\sigma}\right)^2 \quad (6)$$

III. COMPENSATION MARGIN FOR IMPULSE NOISE

Impulse noise is an additive source that is only active for very short intervals in time. Because of its small duty cycle, the average power of impulse noise is much lower than its instantaneous power during active intervals. This results in a large peak-to-average ratio (PAR) - the salient feature of this type of noise. A bit-loading algorithm designed for Gaussian noise sources underestimates the effect of impulses and yields more frequent errors than expected. Impulse noise is a more problematic source of error when it activates frequently with impulse power larger than background noise. In this case, the error events are dominated by the impulse noise and the error rate will be in the order of impulse activation rate. For instance, in ADSL systems, which are prone to impulse interference from AC power switches, the activation rate of the impulse noise is in the order of AC frequency, resulting in an error rate much higher than the target value.

Figure 2 shows a typical error scatter plot when both Gaussian and impulse noise sources are present in the communication channel. This plot shows a QAM cell corresponding to the constellation point at the center. Each marked point in this plot represents a detected data point at the receiver. The distance of these points from the center shows the detection error. The cluster of points at the center has a Gaussian distribution and represents the detection error due to the background noise when there is no impulse in the channel. The outlying points, which form a ring in the scatter plot, consists of detected points received during the active intervals of the impulse noise.

Figure 3 shows a simple noise model that combines the effects of the background Gaussian noise and the impulse noise. The value of impulse noise during active periods is expressed as $re^{j\theta}$, where r is the impulse magnitude and θ is the impulse phase. A simple model for impulse noise assumes a uniform distribution for impulse phase and a Gaussian distribution with an average of μ_I and a standard deviation of σ_I for impulse magnitude.

Although it is possible to derive the parameters of this model from a set of measurements using, for instance, a Maximum-Likelihood approach, the following assumptions simplify the analysis significantly:

- A1 - The impulse noise activation frequency is much higher than the target error rate.
- A2 - The average value of impulse noise amplitude is much larger than the standard deviation of the Gaussian noise.
- A3 - The variation of the impulse amplitude is negligible comparing to the background noise.
- A4 - The impulse noise has a very small duty cycle.

With these assumptions, the outliers in the scatter plot dominate the error events. Therefore, for the sake of error-rate analysis and bit-loading, one can only consider these data points. Given the value of impulse noise, these points have a biased-Gaussian distribution with a mean value equal to the impulse amplitude and a standard deviation identical to that of background Gaussian noise. These two parameters can be measured as the peak and average (standard deviation or root-mean-square) of the error values, respectively.

The compensation margin associated with this biased-Gaussian noise source can be derived as discussed in section II. In this case, the ratio of mean to standard deviation in equation (6) is simply the peak-to-average ratio of the measured error samples. Therefore, the compensation margin for impulse noise is expressed as

$$M_I = \left(1 + \frac{2}{C} PAR_e\right)^2 \quad (7)$$

Fig. 2- The error scatter plot when the communication channel suffers from both Gaussian and impulse noise sources. The cluster of points at the center represents the effect of Gaussian noise when impulse noise is not active. The outer ring shows the error due to both impulse and Gaussian noise sources.

Fig. 3- A noise model that combines the effects of background Gaussian noise and the impulse noise.

where PAR_e denotes the peak-to-average ratio of the of error samples in each dimension.

IV. COMPENSATION MARGIN FOR TIMING PHASE ERROR

Timing mismatch between transmitter and receiver adds a linear phase to the frequency response of the received signal. The error from this phase shift appears as a multiplicative term that effectively rotates the constellation points by an angle equal to the phase error in each tone. Since phase noise is a multiplicative term, the error is not independent of the signal. In fact, phase error grows linearly with the distance of the data points from the center of a constellation. More specifically, a constellation point at distance r with a small phase error of θ exhibits a detection error of $r\theta$.

Figure 4 shows the error scatter plot for a single constellation point on a tone with a relatively strong phase error. Given the value of phase error and the distance of the constellation point from center, the distribution of data points is biased-Gaussian with a mean of $r\theta$ and a standard deviation of background Gaussian noise, σ .

The value of phase error is usually measured through a phase detector as a component of the phase-lock loop. The power of the background Gaussian noise should be derived from the total noise power measurement. The total power of noise, P , is the sum of the background noise power and the power of phase error:

$$P = 2\sigma^2 + \overline{r^2\theta^2} \quad (8)$$

where $\overline{r^2}$ is the signal power (for each tone) and $\overline{\theta^2}$ is the power of phase error.

From equation (6), the compensation margin for phase noise to be applied to the total measured power of noise is expressed as

$$\left(1 + \frac{2\sqrt{2}}{C} \frac{\mu}{\sqrt{P - \overline{r^2\theta^2}}}\right)^2 \left(1 - \frac{\overline{r^2\theta^2}}{P}\right)$$

where the bias, μ , depends on the constellation point and phase error. This compensation factor should be averaged properly over all constellation points and phase values. In order to simplify the analysis, one can conservatively assume that the farthest constellation points with maximum phase error are the dominant cause of error events. In that case

$$\mu = \theta_{\max} r_{\max} / \sqrt{2}$$

and the compensation margin for phase noise is derived as:

$$M_P = \left(\sqrt{1 - \frac{\overline{r^2\theta^2}}{P}} + \frac{2}{C} PAR_r PAR_\theta \sqrt{\frac{\overline{r^2\theta^2}}{P}} \right)^2 \quad (9)$$

where PAR_r and PAR_θ are the peak-to-average ratios of signal (for each tone) and phase error, respectively. This equation shows that the compensation margin grows and then falls as phase noise increases from zero and then dominates the total noise power. Figure 5 shows this behavior when $2PAR_r PAR_\theta = C$. In general, the compensation margin is at its maximum of

$$M_{p-MAX} = 1 + \left(\frac{2}{C} PAR_r PAR_\theta \right)^2$$

when the ratio of total noise to phase noise power is

Fig. 4- The error scatter plot associated with one constellation point when the communication channel suffers from both Gaussian noise and timing phase error.

Fig. 5- Phase compensation margin as a function of the phase noise to total noise power ratio when $2PAR_r PAR_\theta = C$.

$$\frac{P}{\overline{r^2\theta^2}} = 1 + \left(\frac{2}{C} PAR_r PAR_\theta \right)^{-2}$$

V. RESULTS

We studied the effects of impulse noise and phase error in an ADSL system over various loop lengths with a -140 dBm/Hz of AWG background noise. For impulse noise, a setup with dimmer light switch is used as a realistic source of interference, which generates a PAR of more than 5 over many tones. The impulse frequency for this source is twice the AC power frequency. Given a strong impulse power, every single

ADSL superframe will contain a CRC error and the effective data packet rate will drop to zero. To compensate for impulse noise, its presence is first detected when the frequency of outliers in scatter plot is high enough. Once the line is identified as impulsive, the compensation margin of equation (7) is applied to tones with large error PAR.

Figure 6 compares the data rate at physical layer and the effective rate achieved in a file transfer protocol (FTP) both with and without impulse compensation. The bit-loading algorithm targeted a bit-error rate of 10^{-7} . The connection provisioned for 6 dB of target noise margin with a maximum bit rate of slightly higher than 8 Mbps. Impulse compensation reduces the bit rate at physical layer but it also adds enough noise margin to reduce the CRC rate such that the packet transfer rate at upper network layers is significantly improved.

The effect of timing phase error is studied on a 12,000 feet ADSL line with interference from a strong HDSL source. This interferer has significant power at pilot tone that can potentially increase the phase error. This error grows linearly with frequency and its effect is more pronounced on tones with higher SNR. For instance, 1 degree of phase error on a tone with a total SNR close to 32 dB generates far more error than what the Gaussian model predicts. The value of PAR_r in equation (9), which can vary from 1.0 to 1.7 depending on the constellation size, is assumed to be constant at its maximum. The value of PAR_θ , which is measured to be roughly between 2.5 and 3.5, is also fixed at its maximum.

Table I shows that with no compensation margin, the error rate is measured at more than two orders of magnitude higher than the target error rate. Using the phase compensation margin of equation (9), the measured error rate drops to roughly the same level of the target value.

TABLE I. DATE RATE AND BIT ERROR RATE FOR A 12KFT LINE

	Data Rate (Kbps)	BER
No HDSL interferer	4032	9.3×10^{-8}
24 HDSL interferers No Phase Error Compensation	2496	5.7×10^{-6}
24 HDSL interferers With Phase Error Compensation	2144	6.4×10^{-8}

VI. CONCLUSION

This paper presented a method to compensate for a class of non-Gaussian impairments in multicarrier bit-loading. It also presented the application of this method to two specific sources of error: impulse noise and timing phase error. Studies in an ADSL system showed that with simple noise models, this method provides a robust bit-loading which reduces the CRC error significantly and supports a reasonable data packet rate even in presence of severe sources of impairment. Although the problem is studied in the context of DMT in an ADSL system, the results are also applicable to any

Fig. 6- Data rate versus loop length in presence of impulse noise. Solid and dashed lines indicate the bit rate at physical layer and the average FTP throughput, respectively. Stars and circles mark the rates with and without impulse compensation, respectively.

multicarrier system that allows flexible distribution of bits and power among sub-channels.

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