Optimal Quantization and First-Order Approximation in Multi-Frequency Reconstruction

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Introduction: Multi-frequency reconstruction [1] is probably the most widely used method for off-resonance correction. It is one of the few segmented versions [2] of Conjugate Phase Reconstruction [3]. In conjugate phase reconstruction each voxel is demodulated according to the off-resonance frequency in that voxel. This is a computationally intensive algorithm with limited practical applications. Multi-frequency reconstruction, on the other hand, employs a few number of demodulating frequencies. In other words, the field map is quantized to discrete levels and each voxel is demodulated using the quantized value of off-resonance. Conventional method employs a uniform quantization. Here we propose an optimal quantization scheme adapted to the histogram of the field map. For the same image quality, this may reduce the number of demodulations which in turn decreases the reconstruction time. We also employ a spatially linear function to represent the quantized frequencies. This yields a better approximation of the field map.

Method: Starting with the conjugate phase reconstruction, we can represent the image as:

\[ m(\mathbf{r}) = \sum_t S(t) e^{j2\pi f(t) t} e^{j2\pi k_0(t) \cdot \mathbf{r}} \] (1)

Here \( S(t) \) is the MR signal, \( \mathbf{r} = [x, y, z] \) is the 3D position vector, \( k_0(t) \) is the k-space location, and \( f(\mathbf{r}) \) is the field map. Due to the off-resonance term, the above expression cannot be evaluated using an FFT and this makes this algorithm computationally complex. Nonetheless, we can substantially reduce the complexity by quantizing the field map. Namely, we partition the off-resonance range to \( n \) segments and in the spatial region associated with each segment we linearly approximate the field map as \( f(\mathbf{r}) \approx f_i(\mathbf{r}) = f_{0i} + \alpha_i \cdot \mathbf{r} \). Here \( f_{0i} \) is the constant term and \( \alpha_i = [\alpha_{0i}, \alpha_{yi}, \alpha_{zi}] \) is the linear coefficient vector associated with the field map at the \( i \)th region. With this approximation, we can rewrite Eq. 1 as

\[ m(\mathbf{r}) = \sum_t S(t) \left( \sum_{i=1}^{n} e^{j2\pi f_i(\mathbf{r}) t} w_i(f) \right) e^{j2\pi k_0(t) \cdot \mathbf{r}} \]

\[ = \sum_{i=1}^{n} w_i(f) \sum_t S(t) e^{j2\pi f_i(\mathbf{r}) t} e^{j2\pi k_0(t) \cdot \mathbf{r}} \] (2)

Where \( w_i(f) \) are windows in frequency that separate different frequencies. This leads to a significant improvement in the reconstruction time, with no loss of image quality. The number of segmented regions \( n \) is determined by the spatial distribution of the field map.

Results: We have applied the optimal multi-frequency reconstruction to 3D images. A spherical stack trajectory [6] has been used to acquire data. The FOV is 24 x 24 x 6 cm and the voxel size is 1.2 x 1.2 x 1.2 mm. The TR/TE are 60/8 ms. The field map has FOV but half the resolution of the original image sequence has been implemented on a GE 1.5T. Fig. 2 is a comparison between the uniform and optimal quantization. The number of discrete regions is 8 in both methods. Optimal quantization results in sharper images along with the optimal quantized frequencies. The solid line represents a uniform quantization results in dense segmentation in the most frequently used range of off-resonance frequency.

Conclusion: Optimal quantization based on conjugate phase reconstruction is an efficient method for off-resonance correction in multi-frequency reconstruction. It produces sharper images with the same reconstruction time. A first order approximation provides remarkable reduction in computational time with no loss of image quality. This method can be extended to higher order approximations for better accuracy.